Active Fault Tolerant Control For Missing Measurement Problem In A Quarter Car Model With Linear Matrix Inequality Approach

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***Abstract*—In this paper, we start to investigate missing measurement problem in a Quarter car model with Active Fault Tolerant Control (AFTC). AFTC is used to allow the parameters of the controller to be reconfigured in accordance error information obtained online to improve the stability and overall performance of the system when an error occurs. The design is divided into two parts. The first part is designed H-Infinity observer-based Fault Detection Filter (FDF) to generate a residual signal to estimate fault. FDF is designed to minimize the disturbance effect and maximize sensitivity fault. The second is a design fault tolerant control and Fault Compensation to guarantee the stability and performance system from disturbance by ignoring faults. And the function of fault compensation is to minimize effect fault of the system. The main contribution of this research is AFTC proved to solve the missing measurement problem in a Quarter car model. The simulation showed the effectiveness of this method for missing measurement problem which can reduce peak effect until 50% and 2 s faster in steady condition.**

Keywords*—Missing measurement;Active Fault Tolerant Control;Linear Matrix Inequality;Fault Detection Filter;Fault Tolerant Control,;Fault Compensation;Quarter car

I. Introduction

In modern technological system, it is necessary to design a control system that considers features of safety and tolerance of the fault to improve the reliability. Due to the increasing demands on system safety and reliability nowadays, fault detection, estimation and fault tolerant control have attracted great attention from worldwide research communities [11].

Fault Tolerant Control (FTC) has an ability to react the fault by adjusting its activities to the faulty behavior of the plant. In the existing literature, fault divided into sensor fault, actuator fault and process [2], [3]. But problem likes missing measurement also could disturb the performance of the complex system [4]. In the past few years, the control and filtering problems for complex systems with missing measurements problem have been investigated by many researchers. But many literature only studied about filtering, fault tolerant control and fault compensation not resolved [5].

In this paper, we focus to solved missing measurements problem in a quarter car model with Active Fault Tolerant Control (AFTC) with Linier Matrix Inequality (LMI) approach.

II. MATHEMATICAL MODEL OF THE QUARTER CAR MODEL

The model (show in figure 1) is a quarter car with active suspension. The model has two-degree-of-freedom (2 DOF) consist heave and 1 DOF in the tire. The difference about passive and active suspension is a actuator force (F_u) which reduce acceleration body caused road disturbance.

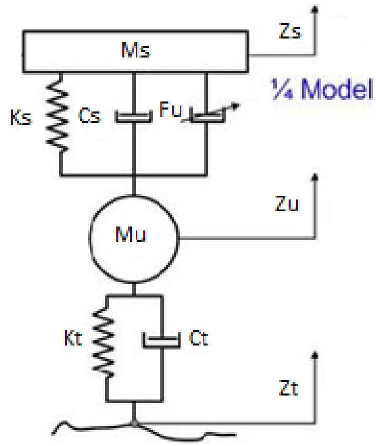

Figure 1 A Quarter Car Model

The mathematical model of the active suspension in a half car model is derived by using Newton's laws of motion as follow:

$$M_s\ddot{Z}_s + K_s(Z_s - Z_u) + C_s(\dot{Z}_s - \dot{Z}_u) - F_u = 0 \tag{1}$$

$$M_u\ddot{Z}_u - K_s(Z_s - Z_u) - C_s(\dot{Z}_s - \dot{Z}_u) + K_t(Z_u - Z_t) + C_t(\dot{Z}_u - \dot{Z}_t) + F_u = 0 \tag{2}$$

where M_u *is* tire mass, M_s *is* sprung mass, $\ddot{Z}_u$ vertical acceleration in the tire, $\ddot{Z}_s$ vertical acceleration in the suspension, $\dot{Z}_u$ vertical velocity in the tire, $\dot{Z}_s$ vertical velocity in the suspension, $\dot{Z}_t$ vertical velocity caused road disturbance, Z_t vertical displacement caused road disturbance, Z_u vertical displacement in the tire, Z_s vertical displacement in the suspension, K_t is coefficient spring in the tire, K_s is coefficient spring in the suspension, C_t is coefficient damper in the tire, C_s is coefficient damper in the suspension, F_u is actuator force.

The equation model of the system will be representation in state space state space model is :

$$\begin{aligned} \dot{x}(t) &= Ax + Bu + B_d w \\ y(t) &= Cx + D_f f + D_v v \end{aligned} \tag{3}$$

where,

$$x = [Z_s \quad Z_u \quad \dot{Z}_s \quad \dot{Z}_u]$$
$$u = F_u$$
$$w = [Z_t \quad \dot{Z}_t]$$

so we have,

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{K_s}{M_s} & \frac{K_s}{M_s} & -\frac{C_s}{M_s} & \frac{C_s}{M_s} \\ \frac{K_s}{M_u} & -\frac{(K_s + K_t)}{M_u} & \frac{C_s}{M_u} & -\frac{(C_s + C_t)}{M_u} \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ 0 \\ -\frac{1}{M_s} \\ 0 \end{bmatrix}, B_d = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \frac{K_t}{M_u} & \frac{C_t}{M_u} \end{bmatrix}$$

we use parameter model from [7].

III. PROBLEM FORMULATION

Consider a linier system with missing measurements problem [5] in equation (4).

$$\begin{aligned} \dot{x}(t) &= Ax + Bu + B_d w \\ y(t) &= Cx + D_f f + D_v v \end{aligned} \tag{4}$$

where,

$$f = -\alpha_f Cx(t) \tag{5}$$

For every $\alpha_f \in \mathbb{R}$ is Bernoulli distributed white sequence taking value on 0 and 1 with

$$\begin{cases} Prob\{\alpha_f = 1\} = u \\ Prob\{\alpha_f = 0\} = 1 - u \end{cases} \tag{6}$$

where $u \in [0,1]$ are known constant [5] . $x \in \mathbb{R}^n$ is the state vector, $y \in \mathbb{R}^n$ is the measurement received by sensor. A, B, B_d, C, D_f and D_v are known matrices with appropriate dimensions. u, w, f, v are input state, which u is controls signals, w is road disturbance, f is missing measurement problem, v is noise sensor.

IV. FAULT DETECTION FILTER AND ESTIMATION DESIGN

In this section, we design an Observer-based Fault Detection Filter (FDF) for missing measurement problem to estimate the faulty. Since (A,C) is detectable, given the following Luenberger observer in equation (7) and (8):

$$\begin{aligned} \dot{\hat{x}} &= A\hat{x} + Bu + L(y - \hat{y}) \\ \hat{y} &= C\hat{x} \end{aligned} \tag{7}$$

where $\hat{x} \in \mathbb{R}^n$ and $\hat{y} \in \mathbb{R}^n$ represent state and output estimation vector. $r(t)$ is the residual signal. Observer gain (L) should be guarantee the stability of the observer.

Filtering error estimation given in equation (11) and (12)

$$e = x - \hat{x} \tag{8}$$

$$\dot{e} = \dot{x} - \dot{\hat{x}} \tag{9}$$

Substitute (11) with (4) and (7), so we have,

$$\dot{e} = (A - LC)e + B_d w - LD_f f - LD_v v \tag{10}$$

Residual generator take the form in equation (11)

$$r(t) = y - \hat{y} \tag{11}$$

From equation (4) and (7), equation (11) rewritten be

$$r = Ce + D_f f + D_v v \tag{12}$$

The design objective of the H_∞ Fault Detection Filter (FDF) is to satisfy following condition such that [8]:

1. $(A - LC)$ is Hurwitz
2. $\|G_{rd(jw)}\|_\infty := \sigma_{max}(G_{rd(jw)}) < \gamma$
3. $\|G_{rf(jw)}\|_- := \sigma_{min}(G_{rf(jw)}) > \beta$

Condition 1 and 2 correspond to the requirement for robust H_∞ estimation, condition 2 represent the effect of disturbances on the residual signal $r(t)$. Condition 3 guarantees a sensitivity of $r(t)$ to $f(t)$ (fault). Since $(A - LC)$ is Hurwitz, from condition 2 we have

$$\|G_{rd(jw)}\|_\infty < \gamma \tag{13}$$

Construct a Lyapunov function, when $(P > 0)$ as

$$V(e) = e^T Pe \tag{14}$$

Given Hamiltonian equation.

$$H = \dot{V}(e) + g(r, d) \tag{15}$$

Substitute (15) with equation (14). So we have,

$$H = \dot{e}^T Pe + e^T P\dot{e} + r^T r - \gamma d^T d \tag{16}$$

To assure that $\|G_{rd(jw)}\|_\infty < \gamma$ then $H < 0$ for all r and d. So we have,

$$\dot{e}^T Pe + e^T P\dot{e} + r^T r - \gamma d^T d < 0 \tag{17}$$

Substitute equation (17), with equation (12) and (13). So we have,

$$\left((A-LC)e+B_d w\right)^T Pe + e^T P\left((A-LC)e+B_d w\right) + (Ce+D_v v)^T(Ce+D_v v) - \gamma(B_d w)^T(B_d w) < 0 \quad (18)$$

From equation (18) we obtain

$$[e^T \quad w^T \quad v^T]\begin{bmatrix}\delta & -PB_d & C^T\\ * & -\gamma I & 0\\ * & * & -\gamma I\end{bmatrix}\begin{bmatrix}e\\ w\\ v\end{bmatrix} < 0 \quad (19)$$

where $\delta = A^T P + PA - PLC - C^T L^T P^T$

With Bounded Real Lemma [9], rewrite equation (19) as

$$\begin{bmatrix}\delta & PB_d & -PLD_v + C^T D_v & C^T\\ * & -\gamma I & 0 & 0\\ * & * & -\gamma I & D_v^T\\ * & * & * & -\gamma I\end{bmatrix} < 0 \quad (20)$$

Solve LMI in equation in (19) which satisfy condition 2. And to satisfy condition 3 we have,

$$\left\|G_{rf(jw)}\right\|_- > \beta \quad (21)$$

Construct a Lyapunov function in (14).

$$V(e) = e^T Pe$$

Given Hamiltonian equation.

$$H = \dot{V}(e) + g(r,f) \quad (22)$$

Substitute (22) with equation (15). And we have,

$$H = \dot{e}^T Pe + e^T P\dot{e} + r^T r - \beta f^T f \quad (23)$$

To assure that $\left\|G_{rf(jw)}\right\|_- > \beta$ then $H > 0$ must hold for all x and d. And we have,

$$\dot{e}^T Pe + e^T P\dot{e} + r^T r - \beta f^T f > 0 \quad (24)$$

Substitute equation (25) with equation (12), and we have

$$\left((A-LC)e - LD_f f\right)^T Pe + e^T P\left((A-LC)e - LD_f f\right) + \left(Ce+D_f f\right)^T\left(Ce+D_f f\right) - \beta\left(D_f f\right)^T\left(D_f f\right) > 0 \quad (25)$$

From equation (26) we obtain

$$[f^T \;\; e^T]\left[\begin{bmatrix}D_f^T D_f - \beta & -D_f^T P^T L^T + D_f^T C\\ * & \delta + C^T C\end{bmatrix}\right]\begin{bmatrix}f\\ e\end{bmatrix} > 0 \quad (26)$$

where $\delta = A^T P + PA - PLC - C^T L^T P^T$

Rewritten equation (27) as following:

$$\begin{bmatrix}D_f^T D_f - \beta & -D_f^T P^T L^T + D_f^T C\\ * & \delta + C^T C\end{bmatrix} > 0 \quad (27)$$

solve LMI in equation (28) which satisfy condition 2.

V. FAULT TOLERANT CONTROL AND FAULT COMPENSATION DESIGN

In this section, we design fault tolerant control and fault compensation to minimize effect disturbance and fault of the system, if fault estimate $\hat{f}(t) = f(t) - \tilde{f}(t)$. And we have,

$$\begin{aligned}u(t) &= u_N(t) + u_c(t) = K_c y + V\hat{f}\\ &= -K\hat{x} - V(y-\hat{y})\\ &= -(K+VC)\hat{x} - VCx + VD_f f\end{aligned} \quad (28)$$

where normal case $u_c = 0$, and faulty case $u_c \neq 0$.

Consider a linier system with missing measurement problem in equation (4)

$$\dot{x}(t) = Ax + Bu + B_d w$$

$$y(t) = Cx + D_f f + D_v v$$

Substitute equation (28) with closed loop system in equation (4). And we have,

$$\dot{x} = (A - BVC)x - (BK + BVC)\hat{x} + BVD_f f + B_d w \quad (29)$$

Substitute equation (28) with Luenberger observer in equation (7), so we have,

$$\dot{\hat{x}} = (LC - BVC)x + (A - BK - BVC - LC)\hat{x} + \left(BVD_f + LD_f\right)f \quad (30)$$

Introduce new state $z = [\dot{x} \quad \dot{\hat{x}}]$ and $\eta = [w \quad f]$, so we can rewrite equation (29) and (30) as,

$$\dot{z} = A_c z + B_c \eta$$

$$y = C_c z \quad (31)$$

where,

$$A_c = \begin{bmatrix}A - BVC & -BK - BVC\\ LC - BVC & (A - BK - LC - BVC)\end{bmatrix}$$

$$B_c = \begin{bmatrix}B_d & BVD_f\\ 0 & BVD_f + LD_f\end{bmatrix}$$

$$C_c = [C \quad 0]$$

Subject of this section is to minimize $\left\|G_{y\eta(jw)}\right\|_\infty$ and guarantee stability system from fault and disturbance. We start to construct a Lyapunov function in equation as:

$$V = z^T Pz \quad (32)$$

to minimize $\left\|G_{y\eta(jw)}\right\|_\infty$ and guarantee stability system from fault and disturbance. We have:

$$\left\|G_{y\eta(jw)}\right\|_\infty < \gamma \quad (33)$$

To assure internal stability. It is assumed that the Lyapunov matrix P is symmetric and positive definite ($P > 0$), given Hamiltonian equation is.

$$H = \dot{V}(x) + g(y, \eta) \tag{34}$$

So we have:

$$H = \dot{z}^T P z + z^T P \dot{z} + y^T y - \gamma \eta^T \eta \tag{35}$$

To assure that $\left\| G_{y\eta(jw)} \right\|_\infty < \gamma$, we have equation (36)

$$\dot{z}^T P z + z^T P \dot{z} + y^T y - \gamma \eta^T \eta < \gamma \tag{36}$$

Substitute equation (36) with equation (31), and we have

$$\begin{aligned}(A_c z + B_c \eta)^T P z + z^T P (A_c z + B_c \eta) \\ + (C_c z)^T (C_c z) - \gamma \eta^T \eta < \gamma\end{aligned} \tag{37}$$

From equation (37) we obtain,

$$\begin{bmatrix} z^T & \eta^T \end{bmatrix} \begin{bmatrix} A_c^T P + P A_c + C_c^T & P B_c \\ * & -\gamma I \end{bmatrix} \begin{bmatrix} z \\ \eta \end{bmatrix} < 0 \tag{38}$$

With Schur complement rewritten equation (39) as:

$$\begin{bmatrix} A_c^T P + P A_c & P B_c & C_c^T \\ * & -\gamma I & 0 \\ * & * & -\gamma I \end{bmatrix} < 0 \tag{39}$$

Introduce new matrix,

$$P = \begin{bmatrix} Y & 0 \\ 0 & Y \end{bmatrix} > 0$$

So equation (39) rewritten as,

$$\begin{bmatrix} \Phi_{11} & \Phi_{12} & Y B_d & Y B V D_f & C^T \\ * & \Phi_{22} & 0 & Y(B V D_f + L D_f) & 0 \\ * & * & -\gamma I & 0 & 0 \\ * & * & * & -\gamma I & 0 \\ * & * & * & * & -\gamma I \end{bmatrix} < 0 \tag{40}$$

where,

$$\Phi_{11} = (A^T - C^T V^T B^T) Y + Y (A - B V C)$$

$$\Phi_{12} = (C^T L^T - C^T V^T B^T) Y - Y B K - Y B V C$$

$$\Phi_{22} = (A^T - C^T K^T B^T - C^T V^T B^T - C^T L^T) Y + Y (A - B K - L C - B V C)$$

VI. RESULT AND SIMULATION

The simulation is presented by using MATLAB. Given road disturbance w with magnitude $0.05\,m$ at the t = 1 s which Fig. 2 has been showed. By using MATLAB (with SeDuMi 1.3 and CVX), we solve LMI in equation (20), (27) (40) and get observer gain (L), controller gain (K), compensator gain (V).

$$L = \begin{bmatrix} 2.655 & 1.015 \\ 4.851 & 32.895 \\ 15.746 & 99.879 \\ -174.82 & 1103.66 \end{bmatrix}$$

$$V = \begin{bmatrix} 17043.32 & -35375.17 \end{bmatrix}$$

$$K = \begin{bmatrix} -15223.54 & 38720.25 & 3309.69 & -931.34 \end{bmatrix}$$

In this simulation, we give example faulty case in one of sensor fault which happen in sprung position sensor. Given fault with $u = 0.3$, $v = 10^{-9}$. Fault will be start from

$t = 1.5\,s$ which the sensor sampling is 0.2 s and then the simulation result from proposed FTC will be compared with H_∞ static output feedback controller.

Figure 3 shows the comparison between fault with and without fault tolerant control (FTC) in sprung position displacement. Proposed FTC can reduce the peak effect from faulty case until 50% which shown in 1.5 seconds and become a steady in 3rd second which 2 s faster than without the FTC.

The fault effect in un-sprung position has been showed in Fig. 4 which give 2 s more longer oscillation if not using FTC, furthermore with proposed FTC can steady in 2nd second. This result can be obtained with RMSE of fault estimation error is 0.005 as Fig. 5 shown.

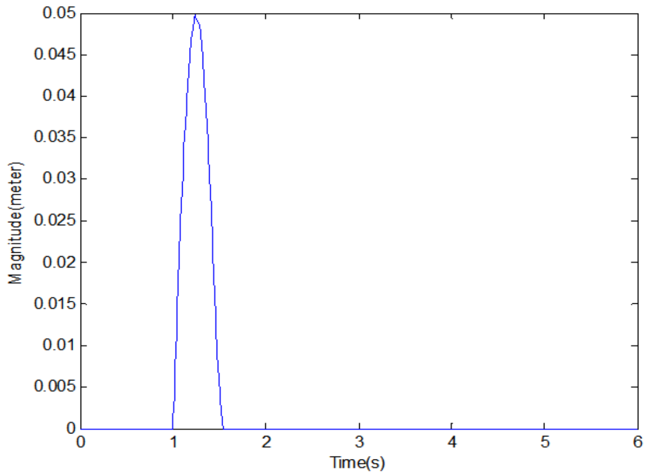

Fig. 1 Road Profile

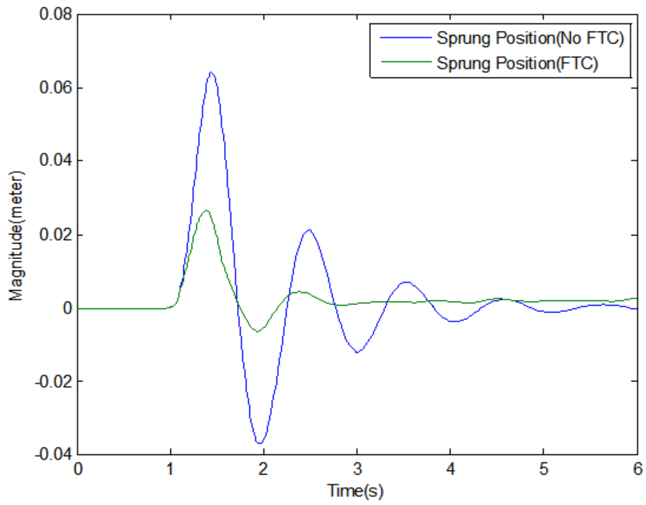

Fig. 2 Comparison with and without FTC in sprung displacement

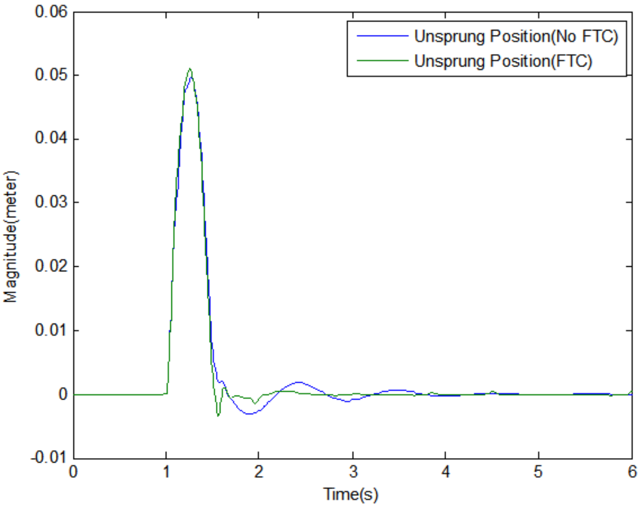

Fig. 3 Comparison with and without FTC in un-sprung displacement

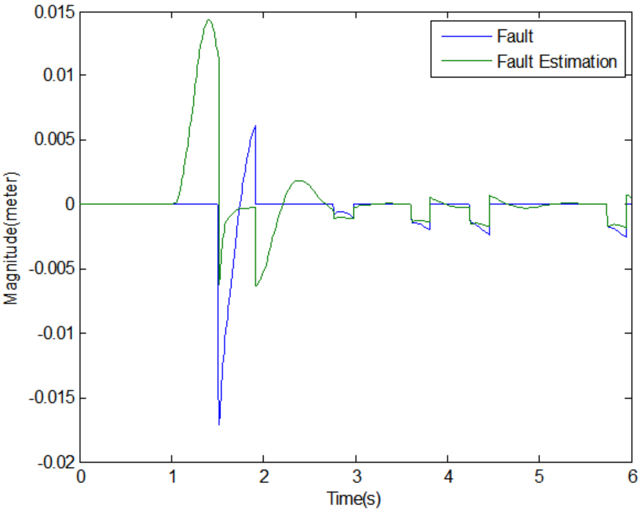

Fig. 4 Estimation Error

VII. CONCLUSION

This research focus on the active fault tolerant control for missing measurement problem in a quarter car model with active suspension system. One of the most important contribution of this paper is solving missing measurement problem with active fault tolerant control which we use Linier Matrix Inequality approach. In this paper, Fault Detection Filter (FDF) works well with a RMSE of estimation error is 0.005. The simulation shows effectiveness of proposed method for the missing measurement problem. The results show, Active Fault Tolerant Control and compensation can reduce displacement peak effect until 50% in system and the missing measurement problem was solved which 2 s faster in steady condition.

References:

[1] Z. Wang, B. Shen, and X. Liu, "Filtering with randomly occurring sensor saturations and missing measurements," *Automatica*, vol. 48, no. 3, pp. 556–562, Mar. 2012.

[2] S. X. Ding and Z. Gao, "Sensor fault reconstruction and sensor compensation for a class of nonlinear state-space systems via a descriptor system approach," *IET Control Theory Appl.*, vol. 1, no. 3, pp. 578–585, May 2007.

[3] J. Zhang, A. K. Swain, S. K. Nguang, and A. Nasiri, "Estimation of actuator and sensor faults for nonlinear systems using a descriptor system approach," in *Decision and Control (CDC), 2014 IEEE 53rd Annual Conference on*, 2014, pp. 1834–1839.

[4] S. M. Savaresi, Ed., *Semi-active suspension control design for vehicles*. Amsterdam [u.a.] Elsevier: Butterworth-Heinemann, 2010.

[5] H. Dong, Z. Wang, and H. Gao, *Filtering, control and fault detection with randomly occurring incomplete information*. Chichester: Wiley, 2013.

[6] S. Sulaiman, P. M. Samin, H. Jamaluddin, R. A. Rahman, and M. S. Burhaumudin, "Modeling and validation of 7-DOF ride model for heavy vehicle," *Proc ICAMME*, 2012.

[7] A. Shirahatt, P. S. S. Prasad, P. Panzade, and M. M. Kulkarni, "Optimal Design of Passenger Car Suspension for Ride and Road Holding," Journal of the Brazilian Society of Mechanical Sciences and Engineering, Vol. xxx, No. 1/75, Januari-March 2008

[8] M. A. Sadrnia, J. Chen, and R. J. Patton, "Robust fault diagnosis observer design using H∞ optimisation and μ synthesis," 1996.

[9] H. Yong, M. Wu, and J. She, "Improved Bounded-Real-Lemma Representation and H∞ Control of Systems With Polytopic Uncertainties," IEEE transactions on Circuit and System, Vol. 52, No. 7, pp. 380-383, July, 2015.

[10] E. Mazar, I. M. Jaimoukha and Z. Li, "An LMI Approach to the Robust Fault Detection and Isolation Problem",

[11] Dian, S., "H-Infinity Filtering Based Fault Detection, Estimation and Fault Tolerant Control", University of Alberta, 2016.